Bond Portfolio and Risk Measurement

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Abstract

This article introduces the importance of measurement of market risk, credit risk and liquidity risk in bond portfolio management. Government bonds are subject only to market risk. Modern risk management uses various methods of computing value at risk as a standard measure for market risk. Corporate bonds are subject to credit risk in addition to interest rate risk. Credit risk measures have been developed in bond portfolio management and become complicated in methodologies, assumptions on statistical distribution, availability of data on exposure, measures of default probabilities and their correlation, and recovery rate. Additionally, the liquidity risk should also be seriously considered and incorporated into the risk model in bond portfolio management.

Introduction

Bond portfolio management is not apparently simple but more complicated than we thought. Fixed income securities have several features that many investors ignore or pay very little attention to their multi facets of risk. Most investors think that investing in fixed income securities, such as bond and corporate debentures, is safe since they could receive coupon payment as their regular income. Fixed income investment has multitude of risks, which not only make the investors lose just some amount of money but sometimes a lot of money, if the securities they invested in become defaults. Typically, there are three major types of risk on bond. The first one is the market risk, which is a potential loss in portfolio value due to undesirable changes in the price-determining factors, caused by the fluctuation in economy and volatility in financial markets, such as interest rates, exchange rates, bond, equity and commodity prices, yield curve, and inflation rate or purchasing power of denominated currency. The second one is credit risk, which is a potential loss in value due to changes of the perceived ability of counter-parties to meet their contractual obligations, for example, credit ratings could be expected to be down graded or the obligor
Market Risk Measurement

As interest rates in the capital market change, individual investor requires yield on bond investment as much as the current rate could provide from the capital market. Portfolio value, as marked to market yields, will be changed due to the market rates change. There are basically two ways to evaluate a bond portfolio value. One way to price a bond is based on individual investor’s required rate of return. Investor is willing to buy a bond at the price determined by his required rate of return. It is simply to apply the required yield \((y)\) to derive the present value of bond \((P)\) with the series of bond future cashflow \((F_t)\), as follows:

\[
P = \sum_{t=1}^{T}[F_t/(1+y/100)^t]
\]

Another way to know the bond price is to discount the bond future cashflows \((F_t)\)’s by zero-coupon yields \((z_t)\’s\), corresponding the time to maturity \((t)\). As a result, the bond price can be written as follows:

\[
P = \sum_{t=1}^{T}[F_t/(1+z_t/100)^t]
\]

Changing in market yields, which affects the bond prices, has a significant impact on a portfolio’s value. A deterministic approach to measure a bond portfolio risk is to find a bond price sensitivity to the change in the discount factors. The traditional measure of price sensitivity is known as duration \((D)\), which evaluates the extent of the price change \((\Delta P)\) for a percentage change in interest rate \((\Delta y)\).

\[
D = - \frac{\Delta P/\Delta y}{P}
\]

However, duration will be a good measure of the change in bond prices if the change in yield is small and market yield is parallelly shifted. However, duration may not capture the effect of change if the yield changes are large, because the relationship between price and yield is not linear. Consequently, convexity, \((C_x)\), is introduced to improve the accuracy of estimation of the percentage change in bond prices. Mathematically, it can be calculated by taking the second derivative of the pricing bond equation and dividing by price:

\[
C_x = \frac{(d^2P/dy^2)}{P}
\]

In practice, we measure the change in yield curve in basis point. So there is another estimator to forecast the sensitivity of a bond price if the yields changed by one basis point \((\Delta z)\). It is referred to as PVBP (present value of a basis point) or DV01 (dollar value of a basis point). Mathematically, the present value of a basis point of each cashflow can be approximated by taking the first derivative of
all bond’s cashflows with a basis point change of zero-coupon yields ($\Delta z_t$’s) as follows:

$$\text{PVBP}_i = \sum_{t=1,T} \Delta P_t(z_t)/\Delta z_t$$

Where $\Delta P_t(z_t)$ represents the change of cashflow’s present value and $\Delta z_t$ is a basis point change of zero-coupon yield at maturity $t$.

Suppose we invest $n$ bonds in a bond portfolio, so its value ($V_p$) is

$$V_p = \sum_{j=1,n} P_j$$

A deterministic approach to measure market risk of a bond portfolio is to derive the portfolio duration ($D_p$), which can be computed by a weighted average ($w_j$) of each bond duration ($D_j$). It can be written as:

$$D_p = \sum_{j=1,n} w_j D_j$$

Since market yields do not fluctuate evenly along the curve, modern concept of measuring risk has been introduced and based on a stochastic movement of the yield curves. It is explained by a statistical method of risk measurement or referred to as Value at Risk or VaR, which is a potential loss due to an adverse effect of the market movement. To understand the concept of VaR, the value of a bond portfolio can be alternatively derived by mapping each bond’s cashflows into the corresponding zero-coupon maturity vertices and sum up their present values. The portfolio value can be written as:

$$V_p = \sum_{t=1,T} \left[ F_t/(1+z_t/100)^t \right]$$

We are interested in the effect of zero-coupon yield change upon the value of the bond portfolio. By Taylor’s first-order approximation, we can have a linear approximation of the portfolio’s profit and loss ($\pi_p$) as:

$$\Delta V_p = \pi_p = \sum_{i=1,T} \left[ \text{PVBP}_i \times \Delta z_i \right] = \sum_{i=1,T} [\delta_i \times \Delta z_i]$$

We can also capture the non-linear risk of the bond portfolio by the Taylor’s second-order approximation, which in addition includes the convexity terms to the profit-and-loss function. The concept of VaR on the measurement of the portfolio market risk will concentrate on the probability distribution of the above portfolio’s profit and loss function. By definition, VaR shows the potential losses ($\Delta V_p$) at a specific confidence level ($\alpha$) over a certain holding period. To calculate VaR, there are 3 basic methodologies for risk manager to choose from, namely Parametric method, Historical Simulation method and Monte Carlo Simulation.

![Figure 1: Definition of Market Value at Risk](image)

$$\text{Prob}(\Delta V_p < \text{VaR}) = \alpha$$

However, all VaR models start with the assumption on changes in market risk factors. Analytical Method, Parametric Method or
**Variance-Covariance Method** estimates VaR by assuming that the market factors having a multivariate Normal distribution. By applying an analytical method in statistics, in practice, the standard deviation of portfolio’s profit and loss is computed. It can be written as

\[
VaR = -z_\alpha \sqrt{\delta^T \Omega \delta}
\]

Where \(\delta\) is the delta vector, which is the present value of a basis point change of zero-coupon yield, \(\Omega\) is the variance-covariance matrix of the zero-coupon yield changes, \([\delta^T \Omega \delta]\) is the variance of portfolio’s profit and loss and \(z_\alpha\) is an inverse function of a standard normal cumulative distribution function with confidence level equal to \(\alpha\).

Calculating VaR under the **historical simulation** method, it assumes that the expected yields in the future would similarly repeat itself as in the historical period, normally for past 250 working days, \(\{\pi_{t-250}, \pi_{t-249}, \ldots, \pi_t\}\). Therefore, the VaR result is computed as the \(p^{th}\)-percentile of ascending sorted series of portfolio’s profit and loss with a corresponding required confidence level, \(\{\pi_{\min}, \ldots, VaR_{\alpha}, \ldots, \pi_{\max}\}\).

Whereas the **Monte Carlo simulation** method is to estimate the VaR by simulating a large number of random scenarios of yield changes, \(\{\Delta z_1, \Delta z_2, \ldots, \Delta z_{10000}\}\), to generate the hypothetical portfolio’s profit and loss \(\{\pi_1, \pi_2, \ldots, \pi_{10000}\}\). Then, the VaR is determined from the distribution of portfolio’s profit and loss at the specified confidence level similar to the historical simulation method.

To ensure that our portfolio position would not eat up our capital under a severe market condition, the approach of **stress testing** and **back testing** are recommended to exercise as supplement to basic VaR measures. For illustration, you may select data from the past historical periods when the market faced unusual events or you can create your own hypothetical situation by changing volatility and correlation factors.

Finally, we will show an idea example to measure a market risk on bond portfolio. For example, risk manager is required to examine the potential loss incurred to his portfolio, consisting of 3 debt securities, namely PTT04NA, LB04NA and SB04NC. Each security consists of 10,000 units. He has a responsibility to report to the risk committee how much the potential loss the portfolio is taking. The date of measurement is hold on December 31, 2003. The portfolio market value is equal to 35.26 million Baht. The risk calculation by the parametric method providing VaR at 95% confidence level with 1-day holding period equal to 8,884 Baht. Therefore, it can be described that after the position closed at the end of business day there would be 95 percent chance to lose not more than 8,884 Baht on the next day.
Covariance Matrix:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>3 month</th>
<th>6 month</th>
<th>1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 month</td>
<td>2.7677</td>
<td>2.2756</td>
<td>2.0586</td>
</tr>
<tr>
<td>6 month</td>
<td>2.2756</td>
<td>2.5035</td>
<td>2.3167</td>
</tr>
<tr>
<td>1 year</td>
<td>2.0586</td>
<td>2.3167</td>
<td>2.6349</td>
</tr>
</tbody>
</table>

Delta Vector: Δ or PVBP

| T or PVBP | 17.76 | 87.63 | 3,236.06 |

Table 1: Market Risk Calculation

<table>
<thead>
<tr>
<th>Sensitivity Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>0.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value at Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence Level</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>95.0%</td>
</tr>
<tr>
<td>97.5%</td>
</tr>
<tr>
<td>99.0%</td>
</tr>
</tbody>
</table>

%VaR(1 d,95%) = -0.0252% P/L Volatility = 5,400.94

Table 2: Market Risk Measures

Credit Risk Measurement

After the economic crisis on July 2, 1997, the bond market in Thailand has not only been expanding tremendously but shown a very high rate of return on investment. Government bonds are subject only to market risk or interest rate risk. However, corporate bonds are subject to credit risk in addition to interest rate risk. Even though credit risk events rarely happened, but once it happens it could have severely damaging impact. Credit risk measurement is the two-stage process: first assigning credit rating to individual obligor and then measuring credit portfolio risk. Credit rating or credit scoring modeling has been widely used in commercial banks. However, the factors and weights of the factors determining credit rating are not disclosed and the details of the rating methodology are proprietary to the banks. Methodology to measure portfolio’s credit risk lagged behind market risk measurement methods in terms of acceptable as a standard, but credit portfolio risk measuring methods are much more complicated than those of the market risk. There are a number of credit risk measurement approaches depending on the availability of data and statistical assumption required by the model.

The technology of risk assessment through corporate ratings has been receiving considerable attention. Credit rating has become a core business for banks and those who invest in corporate bonds. A superior
information and scoring model reflects the performance of commercial banks and fund managers. Scoring models can be characterized by the criteria $i$ to be assessed, and a pre-specified value functions $\{v_i\}$. In other words, the scoring model is characterized by an aggregation rule, which assigns weights $k_i$ to the individual criteria and aggregates these by means of an algebraic rule, usually additive, to form the overall score $V(a)$, e.g., $\{\text{AAA,AA,A,BBB,BB,B,CCC}\}$. The scoring procedure can be written in mathematical terms as:

$$V(a) = \sum_{i=1}^{n} k_i v_i(a_i)$$

The rating procedures based on scoring methods differ in terms of risk-determining factors, rating scales, aggregation rules, and in particular, the choice of weighting factors. The factors, for example, are typically shown in the following figure.

Figure 2: Factors determining credit risk rating

<table>
<thead>
<tr>
<th>Financial Risk</th>
<th>Business Risk</th>
<th>Management Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance sheet and Income statements</td>
<td>Competitive position</td>
<td>• Experience/Succession</td>
</tr>
<tr>
<td>Cashflows and Liquidity</td>
<td>Industrial Sector Evaluation</td>
<td>• Planning and control</td>
</tr>
<tr>
<td>Financial structure (Capital, debt and assets)</td>
<td>Product/brand/diversification</td>
<td>• Organizational structure</td>
</tr>
<tr>
<td>Financial policy and flexibility</td>
<td>Sales Forecasts</td>
<td></td>
</tr>
</tbody>
</table>

Credit Risk Assessment and Rating Scoring Models
- Beaver’s Model
- Bathory Model
- Zscores
- Logit Model
- H-Factor Model
- Artificial Neural Network

Rating Scales
5 risk classes, 10 risk classes, or 16 risk classes $\{\text{AAA,AA,A,BBB,BB,B,CCC}\}$

Ratings of corporate bonds are supposed to be able to reflect likelihood of the obligor will default in the future, in particular, to inform the market about repayment prospects. Investors on fixed-income securities should pay a lot of attention on the ratings of corporate bonds.

Portfolio credit risk involves risk of default as well as the risk of adverse rating change. Considerable work has been done in the US and other developed countries on credit rating migrations. However, there is little work being done in Thailand in this regard. To take an advantage of risk diversification among rating, industries, and other risk attributes, etc., we need to measure credit risk of the portfolio as a whole. As a fund manager of a fixed-income portfolio, we need to know the extent of potential loss if the obligor or the corporate debenture is defaulted or down graded.
The fundamental of portfolio credit risk measure is based on measurement and assumption of the three key variables: potential exposure at default (EAD), probabilities of default (PD), and the loss given default (LGD) or one minus the recovery rate (1-RR). In a two-state default mode, only default and survival events are considered. The credit portfolio expected loss (ELₚ) is equal to the total amount of the product of the single exposures to each one of the n bonds (EADᵢ), the default probabilities (PDᵢ) of each bond, and the losses in case of default (LGDᵢ):

\[
ELₚ = \sum_{i=1}^{n}[PDᵢ * EADᵢ * LGDᵢ]
\]

The above equation can be modified and applied to the bonds with rating exposure. The unexpected loss evaluation takes two stages: at the first stage we calculate unexpected loss, related to every single bond exposure (ULᵢ), to be equal to standard deviation of the loss function. To demonstrate a fundamental of credit risk measurement, we assume that LGDi is a fixed variable (some other model assumes it is a random variable with the beta distribution) and PDᵢ is a binomial random variable. So the unexpected loss depends on the standard deviation of probability of default, \(\sigma(PDᵢ)\), and given by the following formula:

\[
ULᵢ = EADᵢ * LGDᵢ * \sigma(PDᵢ)
\]

\[
= EADᵢ * LGDᵢ * [PDᵢ * (1-PDᵢ)]^{1/2}
\]

In the second stage, the volatility of the whole portfolio potential loss is computed from individual bond loss volatilities, given the known correlation between defaults (default correlation) of the various exposures, \(\rho_ij\):

\[
ULₚ = [\sum_{i=1}^{n}\sum_{j=1}^{n}(\rho_ij * ULᵢ * ULⱼ)]^{1/2}
\]

Figure 3: Definition of Credit Value at Risk

There are several commercial credit risk models such as CreditMetrics, CreditRisk+, KMV Model and Credit Portfolio View, based on different assumptions, probability distribution, estimation techniques on probabilities of default, default correlation, potential exposure at default, and recovery rates. We will not explain these commercial models, but show an application of a simple credit risk model with basic element to the bond portfolio as an illustration.
We show a numerical example to illustrate how a basic credit risk model can be used to evaluate a bond portfolio credit risk. We assume that the portfolio is composed of rated corporate bonds. We estimate the probabilities of default of corporate bond for each rating class from the credit spreads and then find default correlation among rating classes from the time series of defaults. The necessary data needed for calculating the portfolio credit risk is shown in the following tables.

<table>
<thead>
<tr>
<th>Default Correlation between Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
</tr>
<tr>
<td>AAA</td>
</tr>
<tr>
<td>AA</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>BBB</td>
</tr>
<tr>
<td>BB</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>CCC</td>
</tr>
</tbody>
</table>

Table 3: Default Correlation among Credit Rating Classes

<table>
<thead>
<tr>
<th>Elements of Credit Risk of a Corporate Bond Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating Class</td>
</tr>
<tr>
<td>AAA</td>
</tr>
<tr>
<td>AA</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>BBB</td>
</tr>
<tr>
<td>BB</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>CCC</td>
</tr>
<tr>
<td>Total Exposure :=&gt; 300</td>
</tr>
</tbody>
</table>

Recovery Rate 50.0% Expected Loss :=> 0.81
One-year Treasury bond Yield 1.50% Unexpected Loss :=> 10.24

Table 4: Credit Risk Calculation and Measures

Liquidity Risk Measurement
Bond market possess an additional liquidity component that arises from a trader not realizing the price when liquidating his position. The liquidity risk associated with the uncertainty of the spread, particularly for thinly traded under adverse market conditions, is an important part of overall risk and is therefore an important component in risk modeling. The liquidity risk has currently not been able to be quantified as market VaR or credit risks yet. The liquidity of financial market is considered
high if they could be typically characterized by large trading volumes, stable and small bid-ask spreads, stable and high levels of quote depth. The bond portfolio is considered to be liquid depending on sizable position of tradable bond in the market. ThaiBDC’s R&D team has pursued research on bond liquidity and expected that we can build a bond liquidity index as a guideline for investment decision.

Conclusion
Bond portfolio traded in the market is usually marked to market but most investors ignore or pay little attention to credit risk and liquidity risk. Effective bond portfolio management has to understand multitude of risks on bond. We have shown how to compute the risk of bond portfolio in terms of market risk in equation (10) and credit risk in equation (14). We are expecting that the liquidity risk could be measured and incorporated into the standard value-at-risk model which has already taken into account of market risk and unexpected loss for credit risk.

References